Assignment No. 1

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Q-1 A Company manufactures two products A and B The resources are the capacity machine-1, machine-2, machine-3 The available capacities are 50, 25 and 15 resp product A requires 1 hr of machine 2 and 1 hr of machine 3 product B requires 2 hrs of machine-1 2 hrs of machine-2 and 1 hr of machine 3 The profit contribution of Products A and B are RS 5 and RS 4 formulate the linear programming model

Resources	products		Availability
	A	B	
machine-1	–	2	50 unit
machine-2	1	2	25 unit
machine-3	1	1	15 unit
profit	5	4	

LPP Model :- $z = 5x_1 + 4x_2$

Subject to. $2x_2 \le 50$

$x_1 + 2x_2 \le 25$

$x_1 + x_2 \le 15$

$x_1, x_2 \ge 0$

Q-2 An industry is manufacturing two types of product are ₹ 30 & ₹ 40 These two products requires Processing in 3 types of machines the following table shows the available machine hr per day and time required on each machine to produce one kg of A & B, formulate LPP model

Profit/kg	A	B	Total available machine hr/day
machine-1	3	2	600
machine-2	3	5	800
machine-3	5	6	1100.

→ Here product A & B are competing variables & machines are resources. let x_1, x_2 denoted quantity of product A, B resp.

The LPP model

Maximize $Z = 30x_1 + 40x_2$

$$3x_1 + 2x_2 \leq 600$$
$$3x_1 + 5x_2 \leq 800$$
$$5x_1 + 6x_2 \leq 1100$$

$$x_1, x_2 \geq 0$$

Q-3 A Company manufactures two products x & y by using three machines A, B & C machines A, B & C has 4, 24 & 35 hrs of capacity resp available during coming week. one unit of product X requires 1 hr, 3 hr & 10 hrs of machines A, B, C resp. Similarly one unit product Y requires 1 hr, 8 hr & 7 hr of machine A, B & C resp. profit of product x is Rs 5 per product & that of Y is 7 per product formulate LPP model

machines	products		Availability
	X	Y	
A	1	1	4
B	3	8	24
C	10	7	35
Profit	5	7	

$Z = 5x_1 + 7x_2$

LPP equations

$x_1 + x_2 \le 4$

$3x_1 + 8x_2 \le 24$

$10x_1 + 7x_2 \le 35$

Q-4 An organization has 3 machines shops A, B & C & it produces three products X Y & Z using these shops the time available at machine shops A, B, C are 100, 71 & 80 hrs resp. The profit per unit of product X Y & Z is \$ 22, \$ 6 & \$ 2 resp. The table shows time required for each operation for unit amount of each product.

product	A	B	C
X	10	7	2
Y	2	3	4
Z	1	2	1
Available hr	100	72	80

Formulate the linear programming model

→ Decision variables.

Here products X, Y & Z are competing variable and machine A, B, C are available resources let x_1, x_2 & x_3

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denotes no. of unit of product X, Y & Z resp.

The LPP model

Maximize $Z = 22x_1 + 6x_2 + 2x_3$

subject to $10x_1 + 2x_2 + x_3 \leq 100$

$7x_1 + 3x_2 + 2x_3 \leq 72$

$2x_1 + 4x_2 + x_3 \leq 80$

and $x_1, x_2, x_3 \geq 0$

★ Graphical Method

Q-1 Find soln :-

Max $z = 15x_1 + 10x_2$

subject to $4x_1 + 6x_2 \leq 360$

$3x_1 \leq 180$

$5x_2 \leq 200$

$x_1, x_2 \geq 0$

→

$4x_1 + 6x_2 \leq 360$ $\quad 4x_1 + 6x_2 = 360$ — I

x_1	0	90
x_2	60	0

$3x_1 \leq 180 \quad \therefore x_1 = 180/3 = 60$ — II

$5x_2 \leq 200 \quad \therefore x_2 = 200/5 = 40$ — III

Max $z = 15x_1 + 10x_2$

Max $z(A) = (0, 40) = 15(0) + 10 \times (40)$

$= 400$

$\text{Max } z_{(B)} = (30, 40) = 15(30) + 10(40)$

$= 450 + 400 = 850$

$\text{Max } z_{(C)} = (60, 20) = 15(60) + 10(20)$

$= 900 + 200 = 1100$

$\text{Max } z_{(D)} = (60, 0) = 15(60) + 10 \times 0$

$= 900$

$\therefore$ The maximum point is $C(60, 20) = 1100$

$\therefore$ The optimal solution is

$C(60, 20) = 1100$

Q-2 $\text{Min } z = 3x_1 + 2x_2$

Subject to $5x_1 + 2x_2 \geq 10$

$x_1 + x_2 \geq 6$

$x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

$\rightarrow$ $5x_1 + 2x_2 \geq 10$ $\quad\quad$ $5x_1 + 2x_2 = 10$ — I

x_1	0	2
x_2	5	0

$x_1 + x_2 \geq 6$ $\quad\quad$ $x_1 + x_2 = 6$ — II

x_1	0	6
x_2	6	0

$x_1 + 4x_2 \geq 12$ $\quad\quad$ $x_1 + 4x_2 = 12$

x_1	0	12
x_2	3	0

$\text{Min } z = 3x_1 + 2x_2$

$\text{Min } z_{(A)} = (0, 6) = 3(0) + 2(6) = 0 + 12 = 12$

$\text{Min } z_{(B)} = (4, 2) = 3(4) + 2(2) = 12 + 4 = 16$

$\text{Min } z_{(C)} = (12, 0) = 3(12) + 2(0) = 36$

$\therefore$ The minimum value is $A(0, 6) = 12$

$\therefore$ The optimal solution is $\text{Min } (A)(0, 6) = 12$

Simplex Method

Q-1 Find solution using simplex method.

$$\text{Max } z = 30x_1 + 40x_2$$

subject to
$$3x_1 + 2x_2 \le 600$$
$$3x_1 + 5x_2 \le 800$$
$$5x_1 + 6x_2 \le 1100$$
$$\& \quad x_1, x_2 \ge 0$$

$\rightarrow$
$$Z = 30x_1 + 40x_2 + 0s_1 + 0s_2 + 0s_3$$
$$3x_1 + 2x_2 + s_1 = 600$$
$$3x_1 + 5x_2 + s_2 = 800$$
$$5x_1 + 6x_2 + s_3 = 1100$$
$$\& \quad s_1, s_2, s_3 \ge 0 \quad \because \text{slack variables}$$

Basic Table

CBj	Cj	30	40	0	0	0		
	B.V	x_1	x_2	S_1	S_2	S_3	solution	Ratio
0	S_1	3	2	1	0	0	600	$\frac{600}{2} = 30$
0	S_2	3	5	0	1	0	800	$\frac{800}{5} = 16$
0	S_3	5	6	0	0	1	1100	$\frac{1100}{6} = 183.3$
	Z_j	0	0	0	0	0		
	$C_j - Z_j$	30	40	0	0	0		

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First Iteration table

CBj	Cj B.V	X_1	X_2	S_1	S_2	S_3	Solution	Ratio
0	S_1	9/5	0	1	-2/5	0	280	6.22
40	X_2	3/5	1	0	1/5	0	160	10.66
0	S_3	7/5	0	0	-6/5	1	140	4
	Z_j	24	40	0	8	0		
	C_j-Z_j	6	0	0	-8	0.		

Second Iteration table.

CBj	Cj B.V	X_1	X_2	S_1	S_2	S_3	Solution	Ratio
0	S_1	0	0	1	8/7	-9/7	100	
40	X_2	0	1	0	5/7	-3/7	100	
30	X_1	1	0	0	-6/7	5/7	100	
	Z_j	30	40	0	20/7	30/7	7000	
	C_j-Z_j	0	0	0	-20/7	-30/7		

$Z_{max} = 30x_1 + 40x_2$

$= 30 \times 100 + 40 \times 100$

$= 3000 + 4000$

$= 7000$

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Q-2 Find Solution using Big-M Method

$$\text{Min } z = 5x_1 + 3x_2$$

subject to $2x_1 + 4x_2 \le 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \ge 10$. $x_1, x_2 \ge 0$

→ $\text{Min } z = 5x_1 + 3x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

$2x_1 + 4x_2 + S_1 = 12$

$2x_1 + 2x_2 + A_1 = 10$

$5x_1 + 2x_2 - S_2 + A_2 = 10$

Initial Basic table

CBj	Cj	5	3	0	0	M	M	Solution	Ratio
	BV	X_1	X_2	S_1	S_2	A_1	A_2		
0	S_1	2	4	1	0	0	0	12	6
M	A_1	2	2	0	0	1	0	10	5
M	A_2	5	2	0	-1	0	1	10	2
	Z_j	7M	4M	0	-M	M	M		
	$C_j - Z_j$	5-7M	3-4M	0	M	0	0		

NotesSociety

First Iteration table

CBj	Cj	5	3	0	0	M	Solution	Ratio
	BV	X_1	X_2	S_1	S_2	A_1		
0	S_1	0	$\boxed{16/5}$	1	2/5	0	8	2.5
M	A_1	0	6/5	0	2/5	1	6	5
5	X_1	1	2/5	0	-1/5	0	2	5
	Z_j	5	$\left(\frac{6M+2}{5}\right)$	0	$\frac{2M}{5}-1$	M		
	$C_j - Z_j$	0	$\frac{25-6M}{5}$ ↑	0	$-\left(\frac{2M}{5}-1\right)$	0		

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Second Iteration

CBj	Cj	5	3	0	0		Solution	Ratio
	BV	X_1	X_2	S_1	S_2	A		
3	X_2	0	1	5/16	1/8	0	5/2	20
M	A_1	0	0	-3/8	$\boxed{1/4}$	1	3	12
5	X_1	1	0	-1/8	-1/4	0	1	-
	Z_j	5	3	$-\frac{3m}{8}+\frac{5}{16}$	$\frac{M}{4}-\frac{7}{8}$	M		
	C_j-Z_j	0	0	$+\frac{3M}{8}-\frac{5}{16}$	$-\frac{M}{4}+\frac{7}{8}$ ↑	0		

Third Iteration

NotesSociety

CBj	Cj	5	3	0	0	Solution	Ratio
	B.V	X_1	X_2	S_1	S_2		
3	X_2	0	1	1/2	0	1	
0	S_2	0	0	-3/2	1	12	
5	X_1	1	0	-1/2	0	4	
	Z_j	5	3	-1	0	23	
	C_j-Z_j	0	0	1	0		

$\therefore \; C_j - Z_j \geq 0$

$$Z_{max} = 5x_1 + 3x_2$$
$$= 5(4) + 3(1)$$
$$= 20+3$$
$$= 23$$

3) Find solution using Simplex Big-M Method

$\text{Min } z = x_1 + x_2$

Subject to $2x_1 + 4x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$\& \; x_1, x_2 \geq 0$

→ $\text{Min } z = x_1 + x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

$2x_1 + 4x_2 - S_1 + A_1 = 4$

$x_1 + 7x_2 - S_2 + A_2 = 7$

Least +ve

$C_j - Z_j$ -ve

Initial table

CB_j	C_j	1	1	0	0	M	M	Solution	Ratio
	BV	X_1	X_2	S_1	S_2	A_1	A_2		
M	A_1	2	4	-1	0	1	0	4	1
M	A_2	1	$\boxed{7}$	0	-1	0	1	7	1 ←
	Z_j	3M	11M	-M	-M	M	M		
	$C_j - Z_j$	1-3M	1-11M	M	M	0	0		
			↑						

First Iteration

CB_j	C_j	1	1	0	0	M	Solution	Ratio
	BV	X_1	X_2	S_1	S_2	A_1		
M	A_1	$\boxed{10/7}$	0	-1	4/7	1	0	0 ←
1	X_2	1/7	1	0	-1/7	0	1	7
	Z_j	$\frac{10M}{7}+\frac{1}{7}$	1	-M	$\frac{4M}{7}-\frac{1}{7}$	M		
	$C_j - Z_j$	$-\left(\frac{10}{M}+\frac{1}{7}\right)$	0	M	$-\left(\frac{4M}{7}-\frac{1}{7}\right)$	0		

Second Iteration

CBj	cj	1	1	0	0	Solution	Ratio
	BV	x_1	x_2	S_1	S_2		
1	x_1	1	0	$-7/10$	$\boxed{2/5}$	0	0
1	x_2	0	1	$\frac{1}{10}$	$-1/5$	1	—
	z_j	1	1	$-3/5$	$1/5$		
	$c_j - z_j$	0	0	$3/5$	$-1/5$ ↑		

CBj	cj	1	1	0	0	Solution	Ratio
	BV	x_1	x_2	S_1	S_2		
0	S_2	$5/2$	0	$-7/4$	1	0	
9	x_2	$1/2$	1	$-1/25$	0	1	
	z_j	$\cancel{0}\,1/2$	9	$-1/25$		1	
	$c_j - z_j$	$+1/2$	0	$1/25$	0		

Here $c_j - z_j \geq 0$.

$\therefore z_{min} = x_1 + x_2$
$= 0 + 1$
$= 1$

Dual Simplex Method

Q- Find Solution using dual- Simplex method

Min $z = 2x_1 + 3x_2 + 0x_3$

Subject to $2x_1 - x_2 - x_3 \geq 3$ — I

$x_1 - x_2 + x_3 \geq 2$ — II

and $x_1, x_2, x_3 \geq 0$

→ multiply eq^n ① & ② by -1

$-2x_1 + x_2 + x_3 \leq -3$

$-x_1 + x_2 - x_3 \leq -2$

Add slack variable to the equality

Min $z = 2x_1 + 3x_2 + 0x_3 + 0s_1 + 0s_2$

$-2x_1 + x_2 + x_3 + s_1 = -3$

$-x_1 + x_2 - x_3 + s_2 = -2$

CBj	Cj	2	3	0	0	0	Solution
	BV	x_1	x_2	x_3	s_1	s_2	
0	s_1	$\boxed{-2}$	1	1	1	0	-3
0	s_2	-1	1	-1	0	1	-2
	z_j	0	0	0	0	0	
	$c_j - z_j$	2	3	0	0	0	

Determination table.

$-(c_j - z_j)$		-2	-3	0	0	0
	s_1	-2	1	1	1	0
Ratio		1	-3	—	—	—

↑

Iteration I

C_{Bj}	c_j / B.V	2 x_1	3 x_2	0 x_3	0 S_1	0 S_2	Solution
2	X_1	1	$-1/2$	$-1/2$	$-1/2$	0	$3/2$
0	S_2	0	$-1/2$	$\boxed{-3/2}$	$-1/2$	1	$-1/2$
	z_j	2	-1	-1	-1	0	
	$c_j - z_j$	0	4	1	1	0	

Determination table

$-(c_j - z_j)$	0	-4	-1	-1	0
S_2	0	$1/2$	$-3/2$	$-1/2$	1
Ratio	–	-8	$2/3$	2	–

Iteration II

C_{Bj}	c_j / BV	2 X_1	3 X_2	0 X_3	0 S_1	0 S_2	Solution
2	X_1	1	$-2/3$	0	$-1/3$	$-1/3$	$5/3$
3	~~X_2~~	~~0~~	~~$+1/3$~~	~~1~~	~~$1/3$~~		
0	X_3	0	$-1/3$	1	$1/3$	$-2/3$	$1/3$
	z_j	2	$-4/3$	0	$-2/3$	$-2/3$	$10/3$
	$c_j - z_j$	0	$13/3$	0	$2/3$	$2/3$	

$\therefore$ $c_j - z_j \geq 0$ and solution > 0

$\therefore$ $z \min = 2x_1 + 3x_2 + 0x_3$

$= 2\left(\frac{5}{3}\right) + 3(0) + 0 \times \left(\frac{1}{3}\right)$

$= \frac{10}{3}$

$\therefore$ $X_1 = 5/3 \quad X_2 = 0 \quad X_3 = 1/3$

Q-2 Max $z = -15x_1 - 10x_2$

Subject to: $-3x_1 - 5x_2 \le -5$

$-5x_1 - 2x_2 \le -3$

& $x_1, x_2 \ge 0$

→ Max $z = -15x_1 - 10x_2 + 0s_1 + 0s_2$

$-3x_1 - 5x_2 + s_1 = -5$

$-5x_1 - 2x_2 + s_2 = -3$

Basic table

CBj	cj	-15	-10	0	0	
	BV	x_1	x_2	s_1	s_2	Solution
0	s_1	-3	$\boxed{-5}$	1	0	-5
0	s_2	-5	-2	0	1	-3
	z_j	0	0	0	0	
	$c_j - z_j$	-15	-10	0	0	

Determination table

	$c_j - z_j$	-15	-10	0	0	
	s_1	-3	-5	1	0	
	ratio	5	2	-	-	

Iteration I

CBj	cj	-15	-10	0	0	
	BV	x_1	x_2	s_1	s_2	Solution
-10	$\cancel{s_1}$ x_2	3/5	1	-1/5	0	1
0	s_2	$\boxed{-19/5}$	0	-2/5	1	-1
	z_j	-6	-10	2	0	
	$c_j - z_j$	-9	0	-2	0	

Determination table

	$c_j - z_j$	-9	-	-2	-	
	s_2	-19/5	0	-2/5	1	
	ratio	45/19 ↑	-	5	-	

Iteration III

CBj	Cj	-15	-10	0	0	
	BV	x_1	x_2	S_1	S_2	Solution
-10	x_2	0	1	-5/19	3/19	16/19
-15	x_1	1	0	2/19	-6/19	5/19
	Zj	-15	-10	-20/19	-45/19	-235/19
	Cj-Zj	0	0	20/19	45/19	

$\therefore$ $c_j - z_j \geq 0$ and solution > 0

$$\therefore Z_{max} = -15x_1 - 10x_2$$
$$= -15\left(\frac{16}{19}\right) - 10\left(\frac{16}{19}\right)$$
$$= -\frac{75}{19} - \frac{160}{19} = -235/19$$

Q-3 Find solution using two-phase method.

$\text{Max } z = 5x_1 + 8x_2$

subject to $3x_1 + 2x_2 \geq 3$

$x_1 + 4x_2 \geq 4$

$x_1 + x_2 \leq 5$

$\& \; x_1, x_2 \geq 0$

$\rightarrow$ $Z = 5x_1 + 8x_2 + 0S_1 + 0S_2 + 0S_3$

$3x_1 + 2x_2 - S_1 + A_1 = 3$

$x_1 + 4x_2 - S_2 + A_2 = 4$

$x_1 + x_2 + S_3 = 5$

$C_j - Z_j \leq 0$

Most +ve

For phase I Conside $Z = 0x_1 + 0x_2 + 0S_1 + 0S_2 + 0S_3 - A_1 - A_2$

CBj	Cj	0	0	0	0	0	-1	-1		
	BV	x_1	x_2	S_1	S_2	S_3	A_1	A_2	Solution	ratio
-1	A_1	3	2	-1	0	0	1	0	3	3/2
-1	A_2	1	$\boxed{4}$	0	-1	0	0	1	6	1
0	S_3	1	1	0	0	1	0	0	5	5
	Zj	-4	-6	1	1	0	-1	-1		
	Cj-Zj	4	6	-1	-1	0	0	0		

↑

Iteration I

CB_j	c_j	0	0	0	0	0	-1	Solution	Ratio
	BV	x_1	x_2	S_1	S_2	S_3	A_1		
-1	A_1	$\boxed{5/2}$	0	-1	1/2	0	1	1	2/5
0	x_2	1/4	1	0	-1/4	0	0	4	4
0	S_3	3/4	0	0	1/4	1	0	4	16/3
	Z_j	-5/2	0	1	-1/2	0	-1	-1	
	$c_j - Z_j$	5/2	0	-1	1/2	0	0		

Iteration II :

CB_j	c_j	0	0	0	0	0	Solution	Ratio
	BV	x_1	x_2	S_1	S_2	S_3		
0	x_1	1	0	-2/5	1/5	0	2/5	
0	x_2	0	1	1/10	-3/10	0	9/10	
0	S_3	0	0	3/10	1/10	1	37/10	
	Z_j	0	0	0	0	0	0	
	$c_j - z_j$	0	0	0	0	0		

Phase II : Iteration I

Least +ve

CB_j	c_j	5	8	0	0	0	Solution	Ratio
	BV	x_1	x_2	S_1	S_2	S_3		
5	x_1	1	0	-2/5	$\boxed{1/5}$	0	2/5	2/5
8	x_2	0	1	1/10	-3/10	0	9/10	9/10
0	S_3	0	0	3/10	1/10	1	37/10	37/10
	Z_j	5	8	-6/5	-7/5	0	46/5	
	$c_j - z_j$	0	0	6/5	7/5	0		

Iteration II

C_{Bj}	C_j	5	8	0	0	0		
	BV	x_1	x_2	s_1	s_2	s_3	Solution	Ratio
0	s_2	5	0	-2	1	0	2	-1
8	x_2	3/2	1	-1/2	0	0	3/2	-3
0	s_3	-1/2	0	$\boxed{1/2}$	0	1	7/2	7
	Z_j	12	8	-4	0	0		
	$C_j - Z_j$	-7	0	4 ↑	0	0		

Iteration III

C_{Bj}	C_j	5	8	0	0	0		
	BV.	x_1	x_2	s_1	s_2	s_3	Solution	Ratio
0	s_2	3	0	0	1	4	16	
8	x_2	1	1	0	0	1	5	
0	s_1	-1	0	1	0	2	7	
	Z_j	8	8	0	0	8	40	
	$C_j - Z_j$	-3	0	0	0	-8		

Here $C_j - Z_j \le 0$

$\therefore Z_{max} = 5x_1 + 8x_2$

$= 5(0) + 8(5)$

$= 40$

Q-4 Min $Z = 5x_1 + 2x_2 + 10x_3$

Subject to $x_1 - x_3 \le 10$

$x_2 + x_3 \geq 10$

& $x_1, x_2, x_3 \geq 0$.

→ Min $Z = 5x_1 + 2x_2 + 10x_3 + 0s_1 + 0s_2 \cancel{+A} + A_1$

$x_1 - x_3 + s_1 = 10$

$x_2 + x_3 - s_2 + A_1 = 10$

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For phase I Consider $Z = 0x_1 + 0x_2 + 0x_3 + 0S_1 + 0S_2 + A_1$

CBj	Cj	0	0	0	0	0	1		
	BV	x_1	x_2	x_3	S_1	S_2	A_1	solution	Ratio
0	S_1	1	0	−1	1	0	0	10	0
1	A_1	0	$\boxed{1}$	1	0	−1	1	10	10
	Z_j	0	1	1	0	−1	1		
	$C_j - Z_j$	0	−1	−1	0	1	0		

Iteration I

CBj	Cj	0	0	0	0	0		
	BV	x_1	x_2	x_3	S_1	S_2	solution	Ratio
0	S_1	1	0	−1	1	0	10	
0	x_2	0	1	1	0	−1	10	
	Z_j	0	0	0	0	0		
	$C_j - Z_j$	0	0	0	0	0		

Phase II :. Iteration

CBj	Cj	5	2	10	0	0		
	BV	x_1	x_2	x_3	S_1	S_2	Solution	Ratio
0	S_1	1	0	−1	1	0	10	
2	x_2	0	1	1	0	−1	10	
	Z_j	0	2	2	0	−2	20	
	$C_j - Z_j$	5	0	8	0	2		

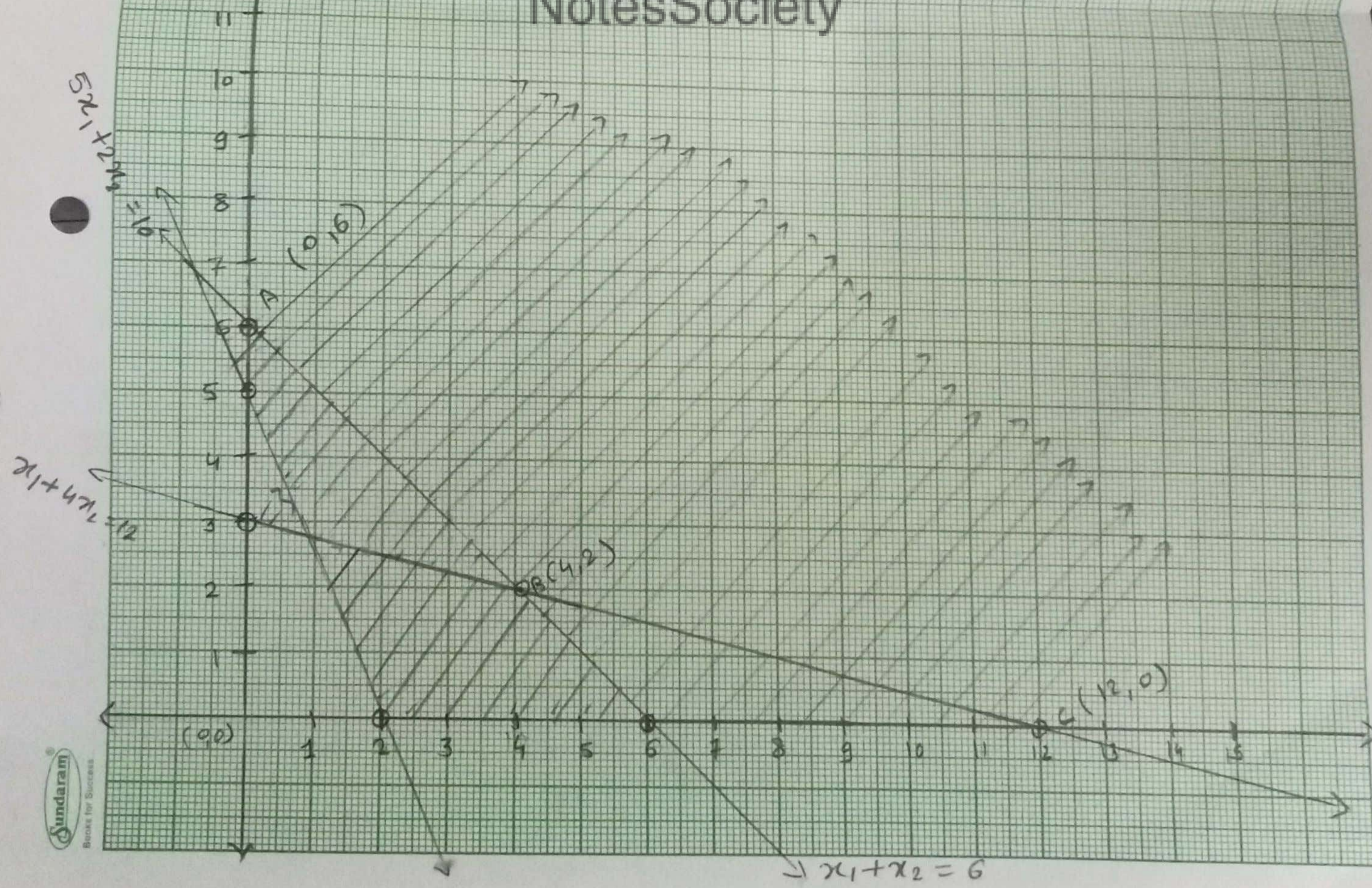
$\therefore C_j - Z_j \geq 0$

$\therefore Z_{min} = 5x_1 + 2x_2 + 10x_3$

$= 5(0) + 2(10) + 10(0)$

$= 20$

Q2.



Q1.

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